

RESEARCH

Open Access



Radio resource management for OFDM-based dual-function radar-communication: sum-rate and fairness

Jia Zhu¹ , Yuanhao Cui^{2*}, Junsheng Mu¹, Zexuan Jing¹ and Xiaojun Jing¹

*Correspondence:
cuiyuanhao@bupt.edu.cn

¹ School of Information and Electronic Engineering, Beijing University of Posts and Telecommunications, Beijing 100876, China

² Department of Electronic and Electrical Engineering, Southern University of Science and Technology, Shenzhen 518055, China

Abstract

This paper focuses on radio resource management (RRM) in multi-user dual-function radar communication (DFRC) systems using orthogonal frequency division multiplexing (OFDM) waveforms. We propose two RRM schemes, one from the perspective of sum rate maximization and the other from the perspective of user fairness maximization. These optimization problems are non-convex due to the presence of mixed integer terms, making them difficult to solve. To address these challenges, we have employed a decomposition approach to transform these two complex problems into separate, more readily solvable ones. In addressing the sum rate maximization problem, we initially introduce a heuristic greedy algorithm to obtain a resource allocation scheme that satisfies radar performance requirements. Subsequently, we utilize a cyclic iterative method along with KKT conditions to solve the sum rate maximization problem for communication users. Concerning the fairness maximization problem for communication users, we similarly employ a heuristic greedy algorithm to obtain a resource allocation scheme that meets radar performance constraints. Then utilize the Lagrangian dual method to solve the multi-user fairness maximization problem for communication users. Our experimental results confirm the effectiveness of the proposed algorithms.

Keywords: Dual-function radar-communication, Fairness, Radio resource management, Waveform design

1 Introduction

The rapid growth in demand for wireless services, coupled with the increasing scarcity of wireless spectrum resources, has resulted in spectrum congestion becoming a critical issue that warrants immediate attention [1]. Expanding the available bandwidth of wireless communication systems has become an urgent issue that requires a solution. The radar frequency band has become the best candidate not only because the radar band has a rich available frequency band, but also because the increase in wireless communication working frequency overlaps with the radar working frequency [2]. Both radio information transmission and radar sensing share similar signal processing methods and system architectures, providing inspiration for their functional integration. The benefits

of this integration are clear: not only can hardware equipment be shared, but frequency bands can also be shared, allowing for efficient use of spectrum resources and alleviating spectrum congestion [3–5].

Radar-communication co-existence (RCC) is one of the implementation schemes, and its purpose is to find an effective interference management method when radar and communication systems coexist in common spectrum. RCC requires two systems to share system parameter information in order to dynamically adjust power allocation, etc. Another mainstream technical solution is DFRC, which is easier to cooperate with radar and communication systems than RCC, and achieves data transmission and remote sensing by using common signals in the same frequency band [6–10].

Multicarrier waveforms have significant advantages in frequency diversity, waveform diversity, and engineering implementation, which make it not only widely used in communication systems but also more and more adopted by radar systems [11, 12]. The transmitter can select and control the required subcarriers at any time instant so that the transmitter can flexibly select the subcarriers according to the channel state information (CSI) to achieve the effect of mitigating interference. From the point of view of the communication system, OFDM is a highly effective physical layer solution that shows great promise, and has been extensively incorporated into wireless industry standards. From a radar perspective, OFDM waveform has many excellent characteristics, such as flexible waveform design [13], high range resolution [14], and high Doppler resolution [15]. Driven by those advantages of the OFDM waveform, many studies regard OFDM as a candidate waveform of DFRC [16–19].

The focus of this study is on radio resource management (RRM) in DFRC system. This topic has been extensively addressed in existing literature, with several proposed solutions for managing radio resources in DFRC systems [20–23]. Experimental results presented by [20] suggest that using target radar cross-section (RCS) as a basis for power allocation can significantly improve the signal-to-noise ratio (SNR) of radar receivers. [21] studied power-saving designs for DFRC systems based on resource allocation. Additionally, [22] proposed an algorithm that performs subcarrier allocation based on channel-to-noise ratio (CNR). Specifically, if the CNR of the radar channel is higher than that of the communication channel, then subcarriers are assigned for radar purpose; conversely, if the CNR of the communication channel is higher than that of the radar channel, then subcarriers are assigned for communication purpose. However, this approach has a notable limitation: when either the radar or communication channel has extremely poor performance, no channels are assigned for either purpose, rendering this strategy impractical. [23] proposed a power allocation algorithm for DFRC system operating in cluttered environments by examining the impact of system clutter on DFRC performance.

In contrast to the RRM strategy employed in single-user DFRC system, the introduction of multi-user OFDM access precipitates more complex issues. Specifically, multi-user OFDM RRM encompasses two intertwined aspects: subcarrier allocation and power control. Subcarrier allocation serves to ascertain whether each subcarrier is designated for use by either the communication or radar function. In the context of this study, this allocation strategy presents a heightened level of challenge. We are confronted with the challenge of determining whether the subcarrier should be allocated

for communication or radar functionalities. If the allocation is for communication, an additional decision must be made regarding the specific communication user to whom the usage should be assigned. Simultaneously, power control is necessitated to fulfill the functional requirements of each user. For instance, the RRM in [24] is formulated as a dynamic resource allocation problem. This problem is resolved using the Lagrangian dual decomposition method to derive the dynamic resource allocation scheme.

While there is a substantial body of literature on RRM on OFDM DFRC, there are still certain aspects that require further attention. Firstly, although numerous research findings exist on the system RRM of communication and radar in the context of a single communication user (CU), the problem of maximizing the sum rate for multiple CUs remains unexplored. When addressing the resource allocation issue, it is also imperative to consider fairness, as algorithms designed to maximize the overall rate often disadvantage users with weaker channel conditions. The pursuit of maximizing the sum rate often comes at the expense of users with poor channel conditions, resulting in significant performance disparities among different users. To tackle this issue, we propose a fairness maximization algorithm to obtain a resource allocation scheme that ensures fairness among CUs.

The main contributions of this study can be summarized as follows:

- This paper delves into the realm of RRM within OFDM DFRC system accommodating multiple CUs. We present two unique optimization challenges in this study: one aimed at enhancing the sum rate, and the other focused on improving fairness. Both of these issues are subject to a variety of constraints, including the radar SNR and the total radiated power of the system. After analyzing and modeling, we formalize these two challenges into two optimization problems. Because they contain mixed-integer variables, both types of problems are non-convex, which usually makes them difficult to solve.
- Inspired by the idea of decomposition [25, 26], we introduce a sum rate maximization optimization strategy. Specifically, we begin by employing a heuristic greedy algorithm to obtain the minimum resource consumption solution that satisfies radar functionality. Subsequently, we maximize the sum rate for multiple CUs based on the remaining resources. To ensure fairness among the multiple CUs, we propose a fairness maximization strategy. Similar to the sum rate maximization strategy, we first solve to obtain the minimum resource consumption that satisfies radar functionality, and then we maximize fairness among CUs based on the remaining resources.
- Our experimental results provide strong evidence that the algorithm introduced in this paper outperforms existing methods. Furthermore, these findings highlight that, under the same radar performance constraint, the fairness-maximizing algorithm willingly compromises a portion of the sum rate to improve the performance of CUs under weaker channel conditions.

2 Methods

In this section, the system model is introduced and the problem parameters are defined. Subsequently, two optimization problems are formulated with the objectives of sum rate and fairness maximization through subcarrier assignment and power allocation.

2.1 System descriptions

We consider a joint active sensing and communication coexistence system, as depicted in Fig. 1. The system employs an OFDM waveform with N subcarriers, sharing the same frequency bandwidth of B Hz and a subcarrier spacing of $\Delta f = B/N$, to enhance spectrum efficiency.

For the communication system, we assume that $c_{n,k}$ represents the symbol of CU k on subcarrier n , with $E\{|c_{n,k}|^2\} = 1$. The communication waveform is denoted by $u_c(t)$, with unit energy, and the carrier frequency is represented by f_c . The signal for the communication service can be represented as follows,

$$x_k(t) = \sum_{n=1}^N c_{n,k} u_c(t) \sqrt{p_n} e^{j2\pi(f_c + n\Delta f)t} \tag{1}$$

where p_n is the power on each subcarrier n , $\mathbf{P} \in \mathbb{C}^{N \times N} = \text{diag}[p_1, p_2, \dots, p_N]$, which is the variable to be optimized. The signal received by the CU k is denoted as,

$$y_k = \sum_{n=1}^N f_{n,k} h_{n,k} x_k + n_k, \tag{2}$$

where $f_{n,k}$ is a binary indicator variable, $h_{n,k}$ is the corresponding gain, n_k is the additive white Gaussian noise (AWGN) with known variance $\sigma_{n,k}^2$.

Assume the CSI can be accurately obtained in advance, which can be done in practice. We choose achievable rate as a measure of communication performance. Specifically, the data transmission rate of CU k can be expressed by

$$R_k = \sum_{n=1}^N f_{n,k} \ln(1 + p_n \gamma_{n,k}), \tag{3}$$

where $\gamma_{n,k} = \frac{h_{n,k}^2}{\sigma_{n,k}^2}$ is the normalized channel gain for communication receiver.

For the radar system, we also use the carrier frequency f_c and subcarrier space Δf , $u_r(t)$ is the unit-energy waveform of radar. The radar transmission signal is represented as,

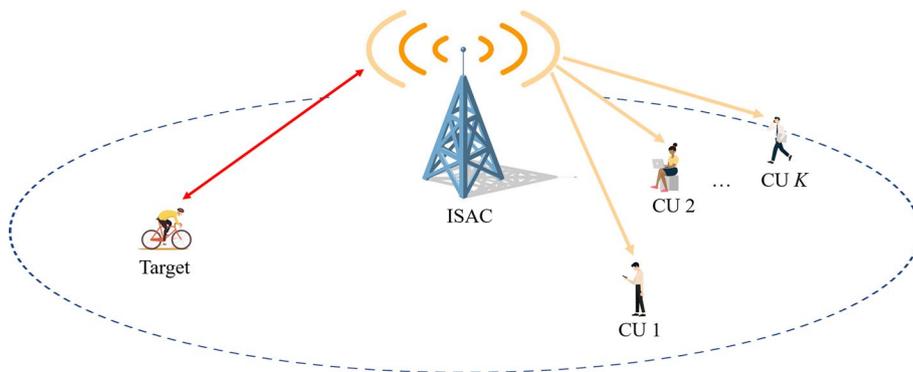


Fig. 1 Simplified system model for multiuser OFDM DFRC

$$x_r(t) = \sum_{n=1}^N u_r(t) \sqrt{p_n} e^{j2\pi(f_c + n\Delta f)t}. \tag{4}$$

The radar received signal is denoted as,

$$y_r = \sum_{n=1}^N (1 - \sum_{k=1}^K f_{n,k}) h_{n,r} x_r + n_r. \tag{5}$$

We choose SNR as the performance measure of the radar system

$$\Theta = \sum_{n=1}^N (1 - \sum_{k=1}^K f_{n,k}) p_n \gamma_{n,r}, \tag{6}$$

where $\gamma_{n,r} = \frac{h_{n,r}^2}{\sigma_{n,r}^2}$ is the normalized channel gain, p_n is the power on subcarrier n .

2.2 Problem formulation

We formulate two optimization problems, one aimed at maximizing the sum rate of CUs (\mathbf{P}_1), and the other aimed at maximizing the rate of CU with the minimum rate (\mathbf{P}_2). The corresponding mathematical formula of \mathbf{P}_1 is presented below:

$$\max \sum_{k=1}^K \sum_{n=1}^N f_{n,k} \ln(1 + p_n \gamma_{n,k}) \tag{7a}$$

$$\text{s.t. } f_{n,k} \in \{0, 1\}, \quad \forall n \in \mathcal{N}, \forall k \in \mathcal{K} \tag{7b}$$

$$\sum_{k=1}^K f_{n,k} \leq 1, \quad \forall n \in \mathcal{N} \tag{7c}$$

$$\Theta \geq \mu, \tag{7d}$$

$$\sum_{n=1}^N p_n \leq P_t, \quad \forall n \in \mathcal{N} \tag{7e}$$

$$0 \leq p_n \leq p_{\max}, \quad \forall n \in \mathcal{N} \tag{7f}$$

Constraints (7b) and (7c) guarantee that each subcarrier can only be allocated to a single user at most. While constraint (7d) guarantees the lowest SNR requirement for radar sensing performance. Constraint (7e) represents the upper limit of the total transmit power of the system, Constraint (7f) represents the limit of power carried by a single subcarrier. In particular, constraint (7f) not only avoids the advantage of losing frequency diversity due to the concentration of transmission power on a small number of subcarriers, but also avoids the interference caused by excessive power of a single carrier by limiting the maximum power carried by a single subcarrier.

Next, we express problem \mathbf{P}_2 as an optimization problem that seeks to maximize the minimum rate among the CUs, thereby ensuring fairness in CUs. The corresponding mathematical formula is provided below:

$$\max \min_{k \in \mathcal{K}} \sum_{n=1}^N f_{n,k} \ln(1 + p_n \gamma_{n,k}) \tag{8a}$$

$$\text{s.t. } (7b) - (7f) \tag{8b}$$

3 The resource allocation algorithms

3.1 Problem analysis

The optimization problem \mathbf{P}_1 is inherently non-convex, primarily due to two main factors. Firstly, the objective function comprises both integer variables $f_{n,k}$ and continuous variables p_n , rendering (7a) non-convex. Secondly, the constraints (7b), (7c), and (7d) are also non-convex in nature. Similarly, the objective function of \mathbf{P}_2 exhibits non-convexity alongside non-convex constraints. Consequently, both (7) and (8) belong to the NP-hard class of problems as established by prior research [27].

Next, two strategies are presented for subcarrier assignment and power allocation to tackle the challenges of maximizing the sum rate and enhancing fairness, respectively.

3.2 Greedy-style heuristic algorithm

Before solving the problems \mathbf{P}_1 and \mathbf{P}_2 , we first analyze the solvability and feasibility of two optimization problems.

For \mathbf{P}_1 and \mathbf{P}_2 , there is no efficient optimal solution. Therefore, we attempt to obtain a suboptimal solution to the optimization problem in a decentralized manner. Upon careful observation, it becomes evident that the feasibility of both \mathbf{P}_1 and \mathbf{P}_2 is contingent upon the SNR of the radar exceeding the threshold μ , given the constraints of P_t and p_{\max} . In the context of the problems this paper seeks to address, the ideal scenario would be one where the radar function operates with minimal resource consumption, thereby allowing the communication function to avail of a larger resource pool. To this end, we introduce a greedy heuristic algorithm designed to satisfy the radar function using the least amount of resources. This approach can be seen as a radar-centric design strategy that prioritizes resource allocation for radar sensing before allocating resources to the CUs. When considering the extreme case where all subcarriers and power are utilized for the radar function, we are faced with the following problem,

$$\min \sum_{n=1}^N p_n \tag{9a}$$

$$\text{s.t. } \sum_{n=1}^N p_n \gamma_{n,r} \geq \mu, \quad \forall n \in \mathcal{N} \tag{9b}$$

$$0 \leq p_n \leq p_{\max}, \quad \forall n \in \mathcal{N} \tag{9c}$$

As (9) is a convex problem, it can be effectively solved using CVX. Upon solving (9), we obtain the result of resource allocation for radar sensing. Therefore, the original problem is decomposed into two sub-problems and a greedy heuristic algorithm is proposed to solve the resource allocation of the radar function.

3.3 For sum-rate maximization

After obtaining the resource allocation solution for radar function, constraint (7d) becomes redundant. Let $\mathcal{N}_c = [1, 2, \dots, N_c]$ denotes the remaining subcarrier used to communication service and $P_c = P_t - P_r$ denotes the remaining total power used to communication service. Then, we rewrite the optimization problem (7) as

$$\max \sum_{k=1}^K \sum_{i=1}^{N_c} f_{i,k} \ln(1 + p_{i,k} \gamma_{i,k}) \tag{10a}$$

$$\text{s.t. } f_{i,k} \in \{0, 1\}, \quad \forall i \in \mathcal{N}_c, \forall k \in \mathcal{K} \tag{10b}$$

$$\sum_{k=1}^K f_{i,k} \leq 1, \quad \forall i \in \mathcal{N}_c \tag{10c}$$

$$\sum_{k=1}^K \sum_{i=1}^{N_c} p_{i,k} \leq P_c, \quad \forall i \in \mathcal{N}_c \tag{10d}$$

$$0 \leq p_{i,k} \leq p_{\max}, \quad \forall i \in \mathcal{N}_c \tag{10e}$$

However, problem (10) remains non-convex. While we can theoretically conduct an exhaustive search for the optimal solution, this method is neither cost-effective nor practical, particularly when dealing with a large number of subcarriers. As a result, there is a need to find a solution that can efficiently solve the problem within polynomial time. To address this challenge, we relax the binary variables $f_{i,k}$ into continuous variables and introduce a penalty term to ensure the optimal solution of the objective function.

$$0 \leq f_{i,k} \leq 1, \quad \forall i \in \mathcal{N}_c, \forall k \in \mathcal{K} \tag{11}$$

Next, we reformulated (10) as

$$\max \sum_{k=1}^K \sum_{i=1}^{N_c} f_{i,k} \ln(1 + p_{i,k} \gamma_{i,k}) + \eta \Phi(f_{i,k}) \tag{12a}$$

$$\text{s.t. } (10c), (10d), (10e), (11). \tag{12b}$$

where $\Phi(f_{i,k}) = \sum_{k \in \mathcal{K}} \sum_{i \in \mathcal{N}_c} (f_{i,k}^2 - f_{i,k}) \leq 0$ called the penalty function. If we set η large enough, $f_{i,k}$ will approach 1 or 0. $\Phi(f_{i,k})$ is a non-convex function. We need to convert it into a convex function to find the subsequent solution. We have

$$\sum_{k \in \mathcal{K}} \sum_{i \in \mathcal{N}_c} f_{i,k}^2 \geq \sum_{k \in \mathcal{K}} \sum_{i \in \mathcal{N}_c} f_{i,k}^{(t)2} + \sum_{k \in \mathcal{K}} \sum_{i \in \mathcal{N}_c} 2f_{i,k}^{(t)} (f_{i,k} - f_{i,k}^{(t)}), \tag{13}$$

where $f_{i,k}^{(t)}$ is the t -th iteration value of $f_{i,k}$. We reformulate the problem (12) as:

$$\max \sum_{k=1}^K \sum_{i=1}^{N_c} f_{i,k} \ln(1 + p_i \gamma_{i,k}) + \eta \tilde{\Phi}(f_{i,k}) \tag{14a}$$

$$\text{s.t.} \quad (c), (10d), (10e), (11) \tag{14b}$$

where $\tilde{\Phi}(f_{i,k})$ is lower bound of $\Phi(f_{i,k})$.

$$\begin{aligned} \tilde{\Phi}(f_{i,k}) = & \sum_{k \in \mathcal{K}} \sum_{i \in \mathcal{N}_c} f_{i,k}^{(t)2} + \sum_{k \in \mathcal{K}} \sum_{i \in \mathcal{N}_c} 2f_{i,k}^{(t)} (f_{i,k} - f_{i,k}^{(t)}) \\ & - \sum_{k \in \mathcal{K}} \sum_{i \in \mathcal{N}_c} f_{i,k} \end{aligned} \tag{15}$$

Hence, We can solve (14) by alternating optimization. By updating the optimization variables $f_{i,k}$ with $p_{i,k}$ fixed, we have the subproblem (16),

$$\max_F \sum_{k=1}^K \sum_{i=1}^{N_c} f_{i,k} \ln(1 + p_{i,k} \gamma_{i,k}) + \eta \tilde{\Phi}(f_{i,k}) \tag{16a}$$

$$\text{s.t.} \quad (10c), (11). \tag{16b}$$

(16) can be efficiently tackled by the BSUM method.

By updating the optimization variables \mathbf{P} with F fixed, we have the subproblem (17),

$$\max_{\mathbf{P}} \sum_{k=1}^K \sum_{i=1}^{N_c} f_{i,k} \ln(1 + p_{i,k} \gamma_{i,k}) \tag{17a}$$

$$\text{s.t.} \quad (10d), (10e). \tag{17b}$$

While (17) can be addressed through optimization tools like CVX, the Lagrange multiplier method presents another viable approach. Leveraging the KKT conditions allows us to derive a closed-form solution for the problem.

$$\begin{aligned} L(\mathbf{P}, \lambda, \lambda^1, \lambda^2) = & - \sum_{k=1}^K \sum_{i=1}^{N_c} f_{i,k} \ln(1 + p_{i,k} \gamma_{i,k}) + \lambda (P_c - \sum_{k=1}^K \sum_{i=1}^{N_c} p_{i,k}) \\ & - \sum_{k=1}^K \sum_{i=1}^{N_c} \lambda_{i,k}^1 p_{i,k} + \sum_{k=1}^K \sum_{i=1}^{N_c} \lambda_{i,k}^2 (p_{i,k} - p_{\max}) \end{aligned} \tag{18}$$

where $\lambda^1 = \sum_{k=1}^K \sum_{i=1}^{N_c} \lambda_{i,k}^1$, $\lambda^2 = \sum_{k=1}^K \sum_{i=1}^{N_c} \lambda_{i,k}^2$ and λ are the Lagrange multipliers for constraints (10e) and (10d). Thus, we can get

$$p_{i,k}^* = \begin{cases} 0, & \frac{f_{i,k}}{\lambda} < \frac{1}{\gamma_{i,k}} \\ \frac{f_{i,k}}{\lambda} - \frac{1}{\gamma_{i,k}}, & \frac{1}{\gamma_{i,k}} < \frac{f_{i,k}}{\lambda} < \frac{1}{\gamma_{i,k}} + p_{\max} \\ p_{\max}, & \frac{f_{i,k}}{\lambda} > \frac{1}{\gamma_{i,k}} + p_{\max} \end{cases} \quad (19)$$

where we can get the value of λ^* by (20).

$$\sum_{k=1}^K \sum_{i=1}^{N_c} p_{i,k}^* \leq p_{\max} \quad (20)$$

We can obtain the value of λ^* through the bisection method, and then use (19) to derive the power allocation results.

We summarize the strategy for maximizing the sum rate as follows: Firstly, we use Algorithm 1 to obtain the minimal resource consumption that satisfies radar functionality. Then, we allocate the remaining resources using Algorithm 2 to achieve the objective of maximizing sum rate.

Algorithm 1 Greedy-style Heuristic Algorithm

Initialization

1. Parameter: $\gamma_{n,r}, p_{\max}, \mu, \tilde{\mu} = 0, z = 1$.
2. Record $\gamma_{n,r}$ sorted in descending order as $\gamma_{n,r,z}$, where the index pair (n, z) is used to distinguish between pre-sort and post-sort subcarriers.

while $\mu \geq \tilde{\mu}$

do

$$\tilde{\mu} = \tilde{\mu} + p_{\max} \gamma_{n,r,z}.$$

$$z = z + 1.$$

Output: $\mathcal{N}_c, \mathcal{N}_s$

Algorithm 2 Sum-Rate Maximization

Input: $\gamma_{n,k}, \gamma_{n,r}, p_{\max}, P_t, \mu$.

Output F, P .

Do

- (1) Based on **Algorithm 1**, obtain the minimal resource allocation scheme that satisfies radar functionality.
 - (2) Obtain $f_{i,k}^*$ by BSUM framework.
 - (3) Obtain $p_{i,k}^*$ by (19).
 - (4) Repeat steps 2 and 3 until convergence
-

3.4 For fairness maximization

In this subsection, we solve the optimization problem \mathbf{P}_2 . First of all, we define an auxiliary variable φ to change the objective function (8a) from non-smooth to smooth. Specifically, (8) is rearranged as

$$\max \quad \varphi \tag{21a}$$

$$\text{s.t.} \quad \sum_{n=1}^N f_{n,k} \ln(1 + p_n \gamma_{n,k}) \geq \varphi, \quad \forall k \in \mathcal{K} \tag{21b}$$

$$(7b)-(7f). \tag{21c}$$

Similar to the approach for solving \mathbf{P}_1 , we still use **Algorithm 1** to obtain a resource allocation scheme that satisfies the radar sensing function. Then we remove the SNR constraint on the radar system (7d) from (21) and obtain the optimization problem (22) of fair resource allocation among CUs.

$$\max \quad \varphi \tag{22a}$$

$$\text{s.t.} \quad \sum_{i=1}^{N_c} f_{i,k} \ln(1 + p_i \gamma_{i,k}) \geq \varphi, \quad \forall k \in \mathcal{K} \tag{22b}$$

$$f_{i,k} \in \{0, 1\}, \quad \forall i \in \mathcal{N}_c, \forall k \in \mathcal{K} \tag{22c}$$

$$\sum_{k=1}^K f_{i,k} \leq 1, \quad \forall i \in \mathcal{N}_c \tag{22d}$$

$$\sum_{i=1}^{N_c} p_i \leq P_c, \quad \forall i \in \mathcal{N}_c \tag{22e}$$

$$0 \leq p_i \leq p_{\max}, \quad \forall i \in \mathcal{N}_c \tag{22f}$$

Constraint (22b) is non-convex set because the coupling between variables $f_{i,k}$ and p_i , which makes the optimization problem (22) intractable. The optimal allocation scheme needs to be obtained through exhaustive search, but this method has a huge amount of calculation and is difficult to apply in practice. Therefore, we propose a suboptimal algorithm with low computational complexity. Specifically, we relax $f_{i,k} \in \{0, 1\}$ to a continuous variable $f_{i,k} \in (0, 1)$ and let $s_{i,k} = f_{i,k} p_i$ as a auxiliary variable. For special case that $f_{i,k} = 0$, we set $f_{i,k} \ln(1 + \frac{s_{i,k} \gamma_{i,k}}{f_{i,k}}) = 0$. We then rearrange (22) as

$$\max \quad \varphi \tag{23a}$$

$$\text{s.t.} \quad \sum_{i=1}^{N_c} f_{i,k} \ln\left(1 + \frac{s_{i,k} \gamma_{i,k}}{f_{i,k}}\right) \geq \varphi, \quad \forall k \in \mathcal{K} \tag{23b}$$

$$\sum_{k=1}^K f_{i,k} \leq 1, \quad \forall i \in \mathcal{N}_c \tag{23c}$$

$$0 \leq f_{i,k} \leq 1, \quad \forall i \in \mathcal{N}_c \tag{23d}$$

$$\sum_{k=1}^K \sum_{i=1}^{N_c} s_{i,k} \leq P_c, \quad \forall i \in \mathcal{N}_c \tag{23e}$$

$$\sum_{k=1}^K s_{i,k} \leq p_{\max}, \quad \forall i \in \mathcal{N}_c \tag{23f}$$

$$s_{i,k} \geq 0, \quad \forall i \in \mathcal{N}_c, \forall k \in \mathcal{K} \tag{23g}$$

$f_{i,k} \ln \left(1 + \frac{s_{i,k} \gamma_{i,k}}{f_{i,k}} \right)$ is a convex function about $(f_{i,k}, s_{i,k})$, so the the feasible region of constraint (23b) is a convex set. The remaining constraints are all convex sets, so the optimization problem (23) is jointly convex about $(f_{i,k}, s_{i,k}, \varphi)$. Problem (23) can be effectively solved by Lagrangian decomposition, so we write its Lagrangian function as

$$\begin{aligned} L(\mathbf{F}, \mathbf{S}, \varphi, \boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\lambda}, \boldsymbol{\chi}) = & \varphi + \sum_{k \in \mathcal{K}} \alpha_k \left(\sum_{i \in \mathcal{N}_c} f_{i,k} \ln \left(1 + \frac{s_{i,k}}{f_{i,k}} \gamma_{i,k} \right) - \varphi \right) + \sum_{i \in \mathcal{N}_c} \beta_i \left(1 - \sum_{k \in \mathcal{K}} f_{i,k} \right) \\ & + \lambda (P_c - \sum_{k \in \mathcal{K}} \sum_{i \in \mathcal{N}_c} s_{i,k}) + \sum_{i \in \mathcal{N}_c} \chi_i (p_{\max} - \sum_{k \in \mathcal{K}} s_{i,k}), \end{aligned} \tag{24}$$

where $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$, $\boldsymbol{\lambda}$ and $\boldsymbol{\chi}$ are the Lagrange multiplier vectors corresponding to (23b), (23c), (23e) and (23f) in (23), respectively. It's dual problem is denoted as

$$G(\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\lambda}, \boldsymbol{\chi}) = \min_{\{\mathbf{F}, \mathbf{S}, \varphi\}} L(\mathbf{F}, \mathbf{S}, \varphi, \boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\lambda}, \boldsymbol{\chi}). \tag{25}$$

By solving problem (25) we can get the solution of the original problem. In detail, we first solve $f_{i,k}$ and $s_{i,k}$ with the Lagrange multiplier fixed, and then to obtain the Lagrange multiplier is according to $f_{i,k}$ and $s_{i,k}$. We first solve for subcarrier and power allocation, reformulating the problem as follows

$$\begin{aligned} \max \quad & \sum_{k \in \mathcal{K}} \alpha_k \sum_{i \in \mathcal{N}_c} f_{i,k} \ln \left(1 + \frac{s_{i,k}}{f_{i,k}} \gamma_{i,k} \right) \\ & - \sum_{i \in \mathcal{N}_c} \beta_i \sum_{k \in \mathcal{K}} f_{i,k} - \lambda \sum_{k \in \mathcal{K}} \sum_{i \in \mathcal{N}_c} s_{i,k} - \sum_{i \in \mathcal{N}_c} \chi_i \sum_{k \in \mathcal{K}} s_{i,k} \\ \text{s.t.} \quad & (23d), (23g). \end{aligned} \tag{26}$$

By fixing $f_{i,k}$ and $s_{i,k}$ respectively, we can get the following formulas,

$$\frac{\partial L(\mathbf{F}, \mathbf{S}, \varphi, \boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\lambda}, \boldsymbol{\chi})}{\partial s_{i,k}^*} = \frac{\alpha_k f_{i,k}}{\lambda + \chi_i} - \frac{f_{i,k}}{\gamma_{i,k}}, \tag{27}$$

$$\frac{\partial L(\mathbf{F}, \mathbf{S}, \varphi, \boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\lambda}, \boldsymbol{\chi})}{\partial f_{i,k}} = \alpha_k \left[\ln\left(1 + \frac{s_{i,k} \gamma_{i,k}}{f_{i,k}}\right) - \frac{s_{i,k} \gamma_{i,k}}{f_{i,k} + s_{i,k} \gamma_{i,k}} \right] - \beta_i, \tag{28}$$

According to constraint (23g), we have

$$p_i^* = \frac{s_{i,k}^*}{f_{i,k}^*} \Big|_{\frac{\partial L(\mathbf{F}, \mathbf{S}, \varphi, \boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\lambda}, \boldsymbol{\chi})}{\partial s_{i,k}^*} = 0} = \left[\frac{\alpha_k}{\lambda + \chi_i} - \frac{1}{\gamma_{i,k}} \right]^+. \tag{29}$$

For $f_{i,k}$, we have

$$\frac{\partial L(\mathbf{F}, \mathbf{S}, \varphi, \boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\lambda}, \boldsymbol{\chi})}{\partial f_{i,k}^*} \begin{cases} < 0, f_{i,k}^* = 0 \\ = 0, 0 < f_{i,k}^* < 1 \\ > 0, f_{i,k}^* = 1 \end{cases} \tag{30}$$

Substiuting (29) into (28) and exploiting (30), we have

$$f_n^* = \begin{cases} 1, X_{i,k} > \beta_i \\ 0, X_{i,k} < \beta_i \end{cases} \tag{31}$$

where

$$X_{i,k} = \alpha_k \left\{ \left[\ln\left(\frac{\alpha_k \gamma_{i,k}}{\lambda + \chi_i}\right) \right]^+ - \left[1 - \frac{\lambda + \chi_i}{\alpha_k \gamma_{i,k}} \right]^+ \right\}. \tag{32}$$

Lagrange multipliers are updated by subgradient method,

$$\alpha_k(t + 1) = [\lambda_k(t) + d_1(t) \Delta \alpha_k(t)]^+, \tag{33}$$

$$\chi_i(t + 1) = [\chi_i(t) + d_2(t) \Delta \chi_i(t)]^+, \tag{34}$$

where

$$\Delta \alpha_k = \sum_{i \in \mathcal{N}_c} f_{i,k}^* \ln \left(1 + \frac{s_{i,k}^*}{f_{i,k}^*} \gamma_{i,k} \right) - \varphi \tag{35}$$

$$\Delta \chi_i = P_c - \sum_{k \in \mathcal{K}} s_{i,k}^*. \tag{36}$$

t is the iteration index and $d_1(t)$ and $d_2(t)$ are positive step sizes. In the actual solution, $\boldsymbol{\lambda}$ and $\boldsymbol{\beta}$ is constant.

Finally, we obtain the optimal φ by solving (37)

$$\max \quad \left(1 - \sum_{k \in \mathcal{K}} \alpha_k \right) \varphi \tag{37a}$$

$$\text{s.t.} \quad 0 \leq \varphi \leq \sum_{i \in \mathcal{N}_c} f_{i,k} \ln\left(1 + \frac{s_{i,k} \gamma_{i,k}}{f_{i,k}}\right). \tag{37b}$$

Its solution is as follows,

$$\varphi^* = \begin{cases} \min_k \sum_{i \in \mathcal{N}_c} f_{i,k}^* \ln(1 + \frac{s_{i,k}^* \gamma_{i,k}}{f_{i,k}^*}), & \sum_{k \in \mathcal{K}} \alpha_k \leq 1 \\ 0. & \sum_{k \in \mathcal{K}} \alpha_k > 1 \end{cases} \quad (38)$$

We summarize the strategy for maximizing fairness as follows: Firstly, we use Algorithm 1 to obtain the minimal resource consumption that satisfies radar functionality. Then, we allocate the remaining resources using Algorithm 3 to achieve the objective of maximizing fairness.

Algorithm 3 Fairness Maximization

Initialization: $t = 0, \rho, \alpha_k, \beta_i, \lambda, \chi_i, t_{\max}, d_1, d_2$.
Do
 1 Based on **Algorithm 1**, obtain the minimal resource allocation scheme that satisfies radar functionality.
Repeat
 2 Calculate power allocation P_n by solving (29).
 3 Calculate subcarrier assignment f_n by solving (31).
 4 Update α_k and χ_i from (33) and (34) .
 $t = t + 1$.
if $\| \alpha_k(t + 1) - \alpha_k(t) \| \leq \rho$ and $\| \chi_i(t + 1) - \chi_i(t) \| \leq \rho$.
 break.
end if.
until : $t > t_{\max}$.
Output $f_{n,k}, p_n$.

3.5 Computational complexity

The computational complexity of Algorithm 1 is given as $\mathcal{Q}(N)$, primarily dictated by the quantity of subcarriers. The computational complexity of Algorithm 2 primarily comprises two components: the first being the complexity $\mathcal{Q}(N_c K)$, associated with the resolution of subcarrier allocation given fixed power variables, and the second being the complexity $\mathcal{Q}(N_c K)$, linked to the determination of power allocation under fixed subcarrier allocation variables. The computational complexity of Algorithm 3 is denoted as $\mathcal{Q}(N_c K t_{\max})$, with t_{\max} representing the maximum iteration count.

4 Results and discussion

To illustrate the effectiveness of the proposed algorithms, we present a multitude of simulation results. We also draw comparisons with two distinct allocation algorithms: the allocation scheme rooted in the greedy algorithm (Greedy) as detailed in [28], and the Subcarrier Assignment Under Uniform Power (SAUP) allocation. We denote the strategy aimed at maximizing sum rate as the Max algorithm and the strategy focused on maximizing fairness as the Max-Min algorithm.

The Greedy algorithm, in its operation, assigns the highest permissible power to subcarriers that exhibit a favorable perceptual channel status, until the radar SNR constraint is fulfilled. The residual subcarriers and power are then distributed among multiple communication users. Conversely, the SAUP algorithm uniformly allocates power to each user, subsequently optimizing only the subcarrier allocation.

Table 1 Simulation parameters

Parameters	Values	Parameters	Values
N	128	B	10 MHz
f_c	1.8 GHz	ρ	0.000001
$\sigma_{n,k}^2$	$1.6e^{-13}W$	$\sigma_{n,r}^2$	$1.6e^{-13}W$
p_{max}	30 W	P_t	2000 W
Cell radius	800 meter	Pathloss model	WINNER II [29]
d_1	0.01	d_2	0.01
ϵ	0.005	μ	30 dB

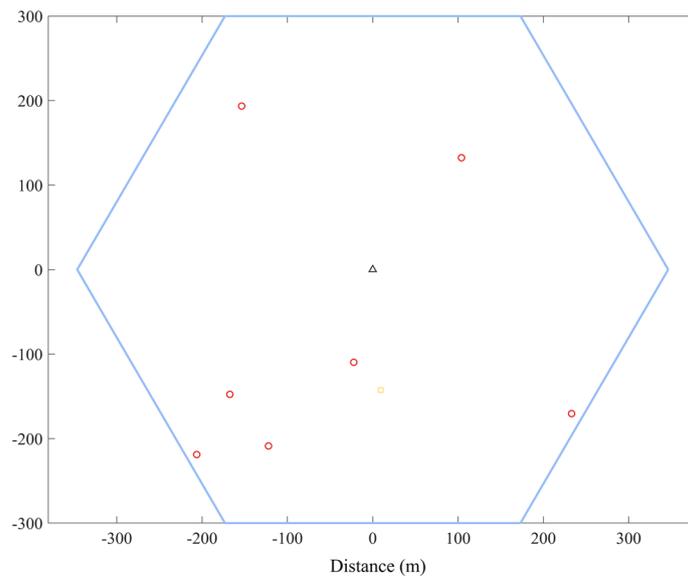


Fig. 2 Distribution of communication users and radar target in experimental scenario

We consider a scenario where the DFRC system is serving one RU and 7 downlink CUs with 128 subcarriers, in a cellular of radius 800m. The specific parameters are set as shown in the Table 1.

We evaluate the fairness of the algorithm according to the reachable rate between CUs. $\mathbf{c} = [c_1, c_2, \dots, c_K]^T$ is the Jain's fairness index vector, and c_k corresponds to the rate of the k th CU,

$$J = \frac{\left(\sum_{k=1}^K c_k\right)^2}{K \sum_{k=1}^K c_k^2} \tag{39}$$

This index, which measures fairness, ranges from a value of $1/K$, indicating a complete absence of fairness, to a value of 1, indicating perfect fairness.

In our numerical experiments, we randomly generated the positions of both CUs and radar target. Subsequently, we generated the channels between each BS-CU pair using the WINNER II path-loss model [29]. A system setup exemplifying a randomly chosen set of CU positions is illustrated in Fig. 2.

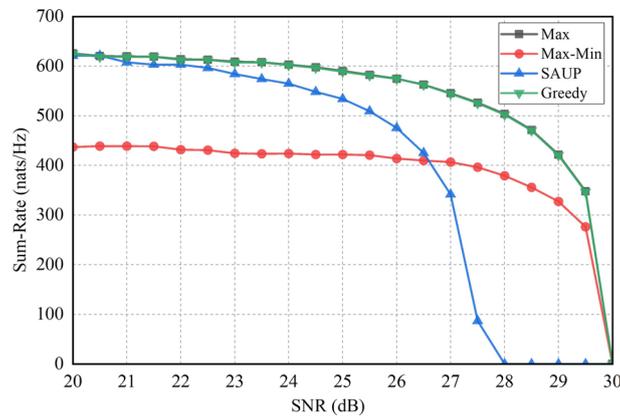


Fig. 3 Sum rate maximization versus SNR with different algorithms

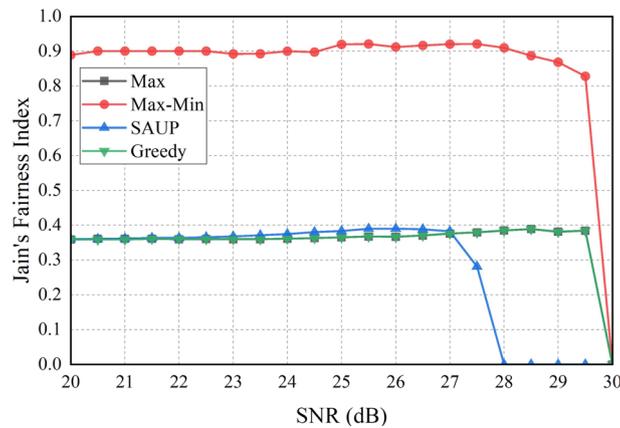


Fig. 4 Fairness maximization versus SNR with different algorithms

Figures 3 and 4 respectively depict the performance characteristics of various algorithms with respect to the maximization of rate and fairness. Insights drawn from Fig. 3 indicate that all aforementioned algorithms exhibit a decreasing function behavior with respect to SNR. Such an outcome is anticipatable. With the escalation of the radar’s SNR, the resources necessitated by the radar functionality proportionally augment, resulting in a contraction of resources accessible for the communication functionality, which subsequently precipitates a decline in the rate. For that purpose, we need to minimize the resources allocated to radar sensing under the constraint of radar minimum SNR. Note also that, the maximum sum rate algorithm outperforms that of the other three algorithms for the whole range of radar SNR evaluated. The Greedy algorithm is slightly lower than maximum sum rate algorithm. Upon the radar’s SNR surpassing 28 dB, the SAUP algorithm encounters failure. This arises from the circumstance where power is uniformly allocated across each subcarrier, thereby rendering it unable to satisfy the stipulated SNR prerequisite. Finally, it can also be seen from the Fig. 3 that the performance of the Max-Min algorithm is worse than the other three algorithms, because it sacrifices the maximum rate of the system in order to ensure fairness among users.

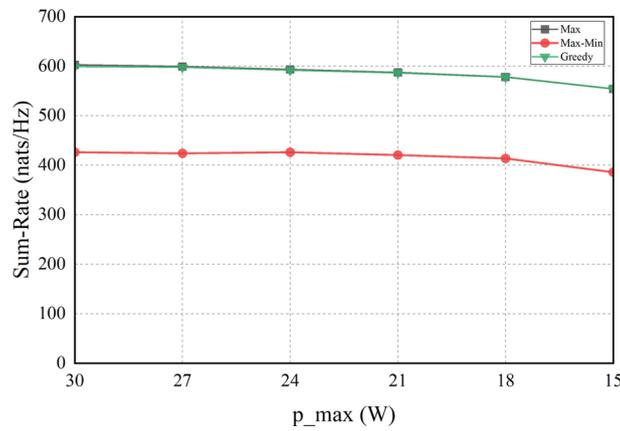


Fig. 5 Sum-rate maximization versus p_{\max} subcarrier power with different algorithms

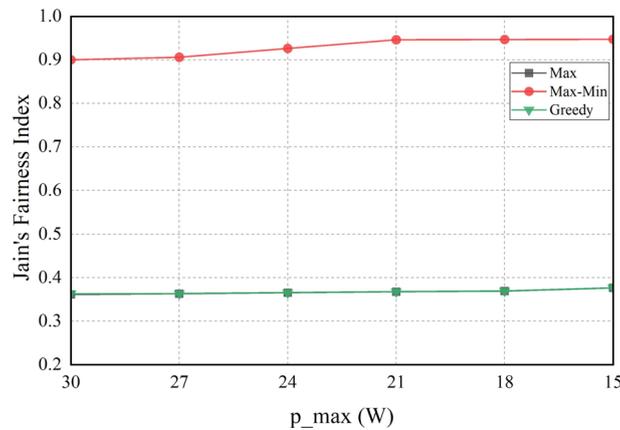


Fig. 6 Fairness maximization versus p_{\max} subcarrier power with different algorithms

As depicted in Fig. 4, the fairness index among users across different algorithms is presented. It is evident that the performance of the maximization algorithm significantly surpasses that of the other three algorithms. The fairness among CUs is often substantially influenced by CUs with inferior channel conditions. Observations reveal that with the escalation of the radar's SNR, there is a marginal enhancement in the fairness among CUs. This phenomenon arises as the radar commandeers an increased share of power and subcarriers, thereby diminishing the resources accessible for CUs. Consequently, the disparity in allocation among users contracts, leading to a slight amelioration in fairness. Nonetheless, when the radar's SNR surpasses a certain threshold, it signifies that the radar functionality has monopolized all resources. As a result, CUs are unable to procure any resources, causing the rate to plummet to zero. This is the underlying cause for the eventual deterioration in performance.

Figures 5 and 6 illustrate the comparisons of transmit power in each subcarrier employing different algorithms for sum rate maximization and fairness maximization, respectively. It can be seen from Fig. 5 that all algorithms are increasing functions of the transmit power that a single carrier can carry. This is because, as the transmit power that can be carried by a single subcarrier increases, subcarriers

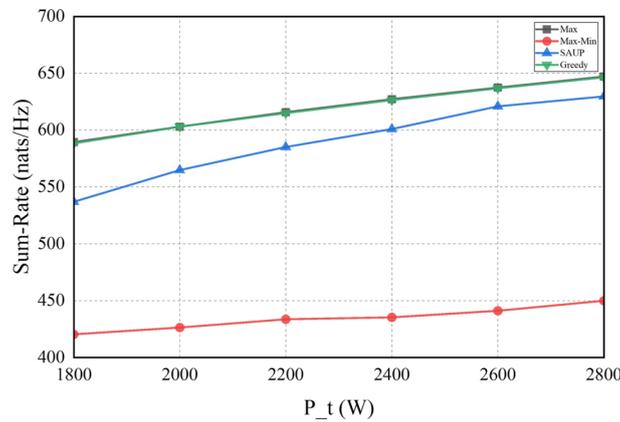


Fig. 7 Sum-rate maximization versus P_t total power with different algorithms

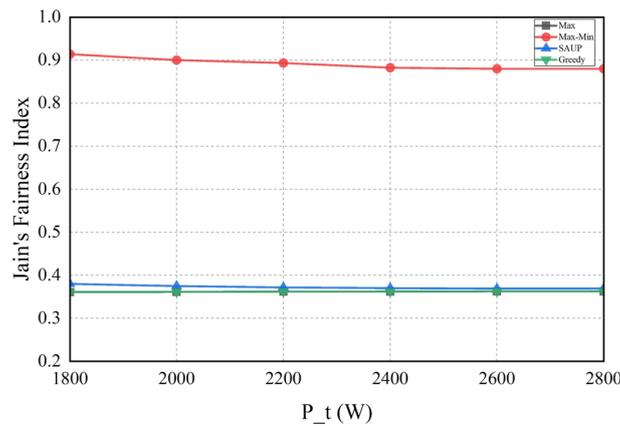


Fig. 8 Fairness maximization versus P_t total power with different algorithms

with good channel status can be allocated more power when the total system power is limited. For fairness among CUs, all algorithms are decreasing functions of the transmit power that a single subcarrier can carry in Fig. 6. The smaller the transmission power that a single subcarrier can carry, the higher the fairness among CUs.

Figures 7 and 8 depict the sum rate and fairness index curves, respectively, as a function of radar SNR for all algorithms. A comparison of the sum rates achieved by the Max, Max-Min, SAUP, and Greedy algorithms when employed in the communication system is presented in Fig. 7. The results indicate that the Max and Greedy algorithms significantly outperform the Max-Min and SAUP algorithms. As shown, when the radar SNR threshold constraint is given, the sum rate increases as a function of the total available power. However, Fig. 8 reveals that increasing the total system power does not result in increased fairness among CUs; rather, fairness decreases slightly as total power increases. This is due to the fact that when the radar SNR constraint is constant and total power increases, the power consumed for radar sensing remains unchanged, leading to an increase in available communication power.

5 Conclusion

In this paper, We have introduced two RRM strategies for DFRC system: one focused on maximizing the sum rate, and the other aimed at maximizing user fairness. Both optimization problems are non-convex and challenging to handle. To address these challenges, we employ a decomposition approach and propose a heuristic greedy algorithm. Initially, we obtain the minimal resource consumption scheme that satisfies radar functionality. Then, we maximize the sum rate on the remaining system resources. In the case of fairness maximization, we similarly employ a heuristic greedy algorithm to determine the minimal resource consumption scheme for the radar system and subsequently maximize user fairness on the remaining system resources. The experimental results demonstrate the effectiveness of our proposed algorithms, which consistently outperform the comparison algorithms.

Our current work is confined to the RRM of DFRC within a single cell, which excludes the more comprehensive RRM scenario involving multiple cells. Our future research will aim to relax this limitation and extend the RRM framework to encompass multiple DFRC cells, where each cell serves multiple users. This extension will introduce challenges related to resource interference and user association between cells. Furthermore, investigating the implications of asymmetric demand among multiple users holds promise as a valuable area for future research.

Abbreviations

OFDM	Orthogonal frequency division multiplexing
RRM	Radio resource management
RCC	Radar-communication co-existence
CSI	Channel state information
DFRC	Dualfunction radar communication
SNR	Signal-to-noise ratio
CNR	Channel-to-noise ratio
RCS	Radar cross-section
CU	Communication user

Author Contributions

This paper is completed with the cooperation of five authors. All authors read and approved the final manuscript.

Funding

None.

Availability of data and materials

The experimental data used and/or analyzed during the current study are available from the corresponding author on reasonable request.

Declarations

Competing interests

The authors declare that they have no competing interests.

Received: 14 July 2023 Accepted: 14 March 2024

Published online: 15 April 2024

References

1. F. Liu, C. Masouros, A.P. Petropulu, H. Griffiths, L. Hanzo, Joint radar and communication design: applications, state-of-the-art, and the road ahead. *IEEE Trans. Commun.* **68**(6), 3834–3862 (2020)
2. H. Griffiths, L. Cohen, S. Watts, E. Mokole, C. Baker, M. Wicks, S. Blunt, Radar spectrum engineering and management: technical and regulatory issues. *Proc. IEEE* **103**(1), 85–102 (2014)

3. Y. Cui, F. Liu, X. Jing, J. Mu, Integrating sensing and communications for ubiquitous IoT: applications, trends, and challenges. *IEEE Netw.* **35**(5), 158–167 (2021). <https://doi.org/10.1109/MNET.010.2100152>
4. F. Liu, C. Masouros, A.P. Petropulu, H. Griffiths, L. Hanzo, Joint radar and communication design: applications, state-of-the-art, and the road ahead. *IEEE Trans. Commun.* **68**(6), 3834–3862 (2020). <https://doi.org/10.1109/TCOMM.2020.2973976>
5. F. Liu, Y. Cui, C. Masouros, J. Xu, T.X. Han, Y.C. Eldar, S. Buzzi, Integrated sensing and communications: toward dual-functional wireless networks for 6g and beyond. *IEEE J. Sel. Areas Commun.* (2022)
6. F. Liu, Y. Cui, C. Masouros, J. Xu, T.X. Han, Y.C. Eldar, S. Buzzi, Integrated sensing and communications: toward dual-functional wireless networks for 6g and beyond. *IEEE J. Sel. Areas Commun.* (2022). <https://doi.org/10.1109/JSAC.2022.3156632>
7. A. Hassanien, M.G. Amin, Y.D. Zhang, F. Ahmad, Signaling strategies for dual-function radar communications: an overview. *IEEE Aerosp. Electron. Syst. Mag.* **31**(10), 36–45 (2016)
8. A. Hassanien, B. Himed, B.D. Rigling, A dual-function mimo radar-communications system using frequency-hopping waveforms. In: 2017 IEEE Radar Conference (RadarConf), pp. 1721–1725 (2017). IEEE
9. A. Hassanien, M.G. Amin, Y.D. Zhang, F. Ahmad, Dual-function radar-communications: information embedding using sidelobe control and waveform diversity. *IEEE Trans. Signal Process.* **64**(8), 2168–2181 (2016)
10. J. Mu, Y. Gong, F. Zhang, Y. Cui, F. Zheng, X. Jing, Integrated sensing and communication-enabled predictive beamforming with deep learning in vehicular networks. *IEEE Commun. Lett.* **25**(10), 3301–3304 (2021). <https://doi.org/10.1109/LCOMM.2021.3098748>
11. S. Sen, A. Nehorai, Adaptive OFDM radar for target detection in multipath scenarios. *IEEE Trans. Signal Process.* **59**(1), 78–90 (2011). <https://doi.org/10.1109/TSP.2010.2086448>
12. M. Bică, V. Koivunen, Generalized multicarrier radar: models and performance. *IEEE Trans. Signal Process.* **64**(17), 4389–4402 (2016). <https://doi.org/10.1109/TSP.2016.2566610>
13. S. Sen, C.W. Glover, Frequency adaptability and waveform design for OFDM radar space-time adaptive processing. In: 2012 IEEE Radar Conference, pp. 0230–0235 (2012). <https://doi.org/10.1109/RADAR.2012.6212142>
14. K. Huo, B. Deng, Y. Liu, W. Jiang, J. Mao, High resolution range profile analysis based on multicarrier phase-coded waveforms of OFDM radar. *J. Syst. Eng. Electron.* **22**(3), 421–427 (2011). <https://doi.org/10.3969/j.issn.1004-4132.2011.03.009>
15. G.E.A. Franken, H. Nikookar, P.V. Genderen, Doppler tolerance of OFDM-coded radar signals. In: 2006 European Radar Conference, pp. 108–111 (2006). <https://doi.org/10.1109/EURAD.2006.280285>
16. C. Sturm, T. Zwick, W. Wiesbeck, An ofdm system concept for joint radar and communications operations. In: VTC Spring 2009 - IEEE 69th Vehicular Technology Conference, pp. 1–5 (2009). <https://doi.org/10.1109/VETECS.2009.5073387>
17. Y.L. Sit, B. Nuss, T. Zwick, On mutual interference cancelation in a MIMO OFDM multiuser radar-communication network. *IEEE Trans. Veh. Technol.* **67**(4), 3339–3348 (2018). <https://doi.org/10.1109/TVT.2017.2781149>
18. Z. Cheng, Z. He, B. Liao, Hybrid beamforming design for OFDM dual-function radar-communication system. *IEEE J. Sel. Top. Signal Process.* **15**(6), 1455–1467 (2021)
19. M.F. Keskin, V. Koivunen, H. Wymeersch, Limited feedforward waveform design for OFDM dual-functional radar-communications. *IEEE Trans. Signal Process.* **69**, 2955–2970 (2021)
20. C. Sahin, P.M. McCormick, J.G. Metcalf, S.D. Blunt, Power-efficient multi-beam phase-attached radar/communications. In: 2019 IEEE Radar Conference (RadarConf), pp. 1–6 (2019). <https://doi.org/10.1109/RADAR.2019.8835583>
21. Y. Zhou, H. Zhou, F. Zhou, Y. Wu, V.C.M. Leung, Resource allocation for a wireless powered integrated radar and communication system. *IEEE Wirel. Commun. Lett.* **8**(1), 253–256 (2019). <https://doi.org/10.1109/LWC.2018.2868819>
22. C. Shi, F. Wang, S. Salous, J. Zhou, Joint subcarrier assignment and power allocation strategy for integrated radar and communications system based on power minimization. *IEEE Sens. J.* **19**(23), 11167–11179 (2019). <https://doi.org/10.1109/JSEN.2019.2935760>
23. F. Wang, H. Li, Power allocation for coexisting multicarrier radar and communication systems in cluttered environments. *IEEE Trans. Signal Process.* **69**, 1603–1613 (2021). <https://doi.org/10.1109/TSP.2021.3060003>
24. H. Yang, Z. Wei, Z. Feng, C. Qiu, Z. Fang, X. Chen, P. Zhang, Queue-aware dynamic resource allocation for the joint communication-radar system. *IEEE Trans. Veh. Technol.* **70**(1), 754–767 (2020)
25. D.P. Palomar, M. Chiang, A tutorial on decomposition methods for network utility maximization. *IEEE J. Sel. Areas Commun.* (2006)
26. R. Zhang, S. Cui, Cooperative interference management with MISO beamforming. *IEEE Trans. Signal Process.* (2010)
27. J.V.C. Evangelista, Z. Sattar, G. Kaddoum, A. Chaaban, Fairness and sum-rate maximization via joint channel and power allocation in uplink SCMA networks. *IEEE Trans. Wirel. Commun.* (2018)
28. F. Wang, H. Li, M.A. Govoni, Power allocation and co-design of multicarrier communication and radar systems for spectral coexistence. *IEEE Trans. Signal Process.* **67**(14), 3818–3831 (2019). <https://doi.org/10.1109/TSP.2019.2920598>
29. Y.D.J. Bultitude, T. Rautiainen, Ist-4-027756 winner ii d1. 1.2 v1. 2 winner ii channel models. EBITG, TUI, UOULL, CU/ CRC, NOKIA, Technical Report (2007)

Publisher's Note

Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.